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AUTHOR Gullickson, Arlen R.; And Others

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ABSTRACT

This study is an analysis of the robustness of the Box-Tiao integrated moving averages model for analysis of time series quasi experiments. One of the assumptions underlying the Box-Tiao model is that all N values of alpha subscript t come from the same population which has a variance sigma squared. The robustness was studied only in terms of homogeneity of variance. The variances of the alpha subscript t values of n subscript 1 observations before the treatment event T and the variances of the alpha subscript t values for the n subscript 2 observations after event T were systematically varied so as to simulate realistic conditions under which the variances of the alpha subscript t would be heterogeneous. The actual percent of t-ratios exceeding 1-alpha superscript t N-2 for these cases was found when the null hypothesis was true. The results of this empirical study were then compared with the percent of t-ratios exceeding 1-alpha superscript t N-2 values for the null hypothesis with homogeneous variances. The trends visible from the results lead to the conclusion that if possible heterogeneity of error variance is suspected then conservitive nominal significance levels should be set if the IMA model is to be used in determining the effect of a treatment. (Author)

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Violation of Homogeneity of Variance Assumption in the Integrated Moving Averages Time Series Model

Arlen R. Gullickson

Larry R. Nelson

Gene V Glass

Laboratory of Educational Research
University of Colorado

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Violation of Homogeneity of Variance

Assumption in the Integrated Moving

Averages Time Series Model

The Time Series Quasi-Experiment is a method for evaluating the change in level between two successive points in a time series. Observations z_t are taken at equally spaced time intervals and one wishes to make inferences about a possible abrupt shift in level of direction or drift of the time series associated with the occurrence of the introduction of an event at a point in time. Donald T. Campbell and Julian C. Stanley presented this Interrupted Time Series Design in Chapter 5 "Experimental and Quasi-experimental Designs for Research on Teaching" in the Handbook of Research on Teaching (1963).

Diagrammatically, the design of the time-series quasi-experiment is as follows: $z_1, z_2 \ldots z_{n_1} \quad T \quad z_{n_1+1}, \ldots z_{n_1+n_2}$. Where z_j represents the jth observation of a variable and T represents the "treatment."

If the trend of the pre-T observations is altered sharply by the introduction of T, we will attribute the alteration (whether a change in level or in direction of drift) to T. A particularly important problem is to determine whether the activity of the time-series near T indicates a genuine effect of T or merely an orderly continuation of the time-series. The problem is "particularly important" because the inferential statistical intuitions of social scientists seem seldom to have been developed on non-independent observations (such as those in most time-series). Hence, statistical significance tests are necessary overseers of one's "considered impressions" of the data.

Box and Tiao (1965) developed a method of evaluating the change in level between two successive points of a non-stationary time-series. Observations z_t are taken at equally spaced points in time and inferences are to be made about a possible shift in level of the time-series associated with the occurrence of the event T. This method appears to be the most suitable method now available for analyzing the time-series quasi-experiments. It has been used as a method of analysis in several published studies. Two studies of note are: "Analysis of the Connecticut speeding crackdown as a time-series quasi experiment" by Gene V Glass in the August 1968 Law and Society Review, and "Analysis of data on the 1900 revision of the German divorce laws as a quasi-experiment" by Gene V Glass, George C. Tiao, and Thomas O. Maguire, Law and Society Review (in press).

The model underlying the Box-Tiao analysis of change in level of a time-series is the integrated moving averages (IMA) model. Essentially the model implies that the system is subjected to periodic random shocks (with zero mean). The initial impact of these shocks on the system is noted as a_t. Some proportion ϕ of these shocks remains in the system and has a positive or negative effect on the system over time, consequently $-1 < \phi < 1$. In terms of these random shocks, the difference between the value of two observations, one at time t, the other at time t-1, may be written as

$$z_t - z_{t-1} = a_t - \phi a_{t-1}.$$

This equation may be solved for z_t as a function of the a's alone.

In order to facilitate solution for z_t, two operators are employed in the following equations; they are the backward shift operator B, which is defined as $Bz_t = z_{t-1}$, hence $B^m z_t = z_{t-m}$; and the backward difference operator ∇ which can be written in terms of B since $\nabla z_t = z_t - z_{t-1} = (1-B)z_t$.

In turn, ∇ has for its inverse the summation operator S given by

$$\nabla^{-1} z_t = S = \sum_{j=0}^{\infty} z_{t-j} \tag{0.1}$$

$$= z_t + z_{t-1} + z_{t-2} + \ldots$$

$$= (1 + B + B^2 + B^3 + \ldots)z_t$$

$$= (1 - B)^{-1} z_t$$

The IMA equation written in operator notation then is:

$$\nabla z_t = (1 - \phi B)\ z_t$$

There is some advantage in writing the right side of the equation in terms of ∇ instead of B.

$$(1 - \phi B) = (1 - \phi)B + (1 - B) = (1 - \phi)B + \nabla = \gamma B + \nabla$$

where $\gamma = 1 - \phi$, and therefore $0 < \gamma < 2$. Substituting into the equation

$$\nabla z_t = (\gamma B + \nabla)\alpha_t$$

$$\nabla z_t = \gamma\,\alpha_{t-1} + \nabla\alpha_t$$

$$z_t = \nabla^{-1}\ (\gamma\alpha_{t-1} + \nabla\alpha_t)$$

$$z_t = \nabla^{-1}\gamma\alpha_{t-1} + \alpha_t$$

from equation (0.1) $\qquad \nabla^{-1}\alpha_t = \sum_{j=0}^{\infty} \alpha_{t-j}$

Therefore $\qquad z_t = \gamma \sum_{j=1}^{\infty} \alpha_j + \alpha_t$

If we express the model in terms of α's entering the system after the time origin k we obtain

$$z_t = L + \gamma \sum_{j=1}^{t-k} \alpha_{t-j} + \alpha_t$$

in which the constant, L, is the value of the system at the origin time k.

By setting $k = 0$, the model's equation can be written using the following notation:[1]

$$z_t = L + \gamma \sum_{i=1}^{t-1} \alpha_i + \alpha_t.$$

Thus the first observation recorded would be $z_1 = L + \alpha_1$, and for the n_1 observations prior to the introduction of a Treatment T

$$z_t = L + \gamma \sum_{i=1}^{t-1} \alpha_i + \alpha_t \qquad (1)$$

for the n_2 observations following T

$$z_t = L + \delta + \gamma \sum_{i=1}^{t-1} \alpha_i + \alpha_t \qquad (2)$$

where:

- z_t is the value of the variable observed at time t,
- L is a fixed but unknown location parameter,
- γ is a parameter descriptive of the degree of interdependence of the observations in the time-series and takes values $0 < \gamma < 2$,
- α_t is a random normal deviate with mean 0 and variance of σ^2,
- δ is the change in level of the time-series caused by T.

Data which conform to the model in (1) and (2) are such that the graph of the time-series follows an eratic, somewhat random path with slight, but no systematic drifts, trends, or cycles. Data which show a systematic increase or decrease over time--such as population and various growth curves--violate the assumption of zero mean for the random variable α_t. For generality, the random variable portion of the model can be allowed to assume an expected value other than zero; thus "drifting" time-series--those showing a constant rise or fall over time--can be accommodated. The generalization of the model in (1) and (2) is called the "integrated moving average model with deterministic drift"[2] and takes the following form:

$$z_1 = L + \beta_1 \text{ and } z_t = L + \gamma \sum_{i=1}^{t-1} \beta_{i-1} + \beta_t, \tag{3}$$

for the $\underline{n}_1$ observations prior to the introduction of T, and

$$z_t = L + \delta + \gamma \sum_{i=1}^{t-1} \beta_{i-1} + \beta_t, \tag{4}$$

for the $\underline{n}_2 = \underline{N} - \underline{n}_1$ observations following T,

where L, γ and δ are interpreted as in the model in (1) and (2), but now $\underline{\beta}$ is a normal variable with variance σ^2 and mean equal to μ.

The parameter μ describes the rate of ascent or descent of the time-series.

It is illuminating to express β as $\mu + a$ and manipulate (3) into a form similar to (1):

$$z_t = L + \mu\gamma(t-1) + \mu + \gamma \sum_{i=1}^{t-1} a_{i-1} + a_t \tag{5}$$

One sees by inspection of (5) that the time-series in (3) will be expected to have "drifted" $\mu\gamma t$ units at time $\underline{t}$.

This model can again be modified so that a parameter descriptive of a change in μ, the drift of the series, is incorporated. It is then possible to estimate all of the parameters in the model for a given value of γ and to test hypotheses about each.

Let z_t denote the observation of a series at time $\underline{t}$, prior to the introduction of a treatment $\underline{T}$:

$$z_t = L + \gamma\mu\,(t-1) + \mu + \gamma \sum_{j=1}^{t-1} a_j + a_t, \tag{3}$$

where the interpretation of the elements of the model are identical to their interpretation given earlier in this paper. The following model is descriptive of the behavior of the series for the $\underline{n}_2$ observations following the introduction of T:

$$z_t = L + \gamma\mu\,(t - 1) + \mu + \gamma\Delta\,(t - n_1 - 1) + \Delta + \gamma \sum_{i=1}^{t-1} a_i + a_t + \delta \qquad (4)$$

Where δ is the change of level of the series between time n_1 and $n_1 + 1$, and Δ is the change in the drift of the series between these two times. Prior to $\underline{T}$, the series drifts (on the average) at a rate of $\gamma\mu$ units (up or down depending on the sign of μ) for each unit of time; after $\underline{T}$, the series drifts $\gamma(\mu + \Delta)$ units on the average for each unit of time.

Interest in this model generally centers on obtaining estimates of the parameters δ and Δ. In order to do this, a collection of $\underline{n}_1 + \underline{n}_2$ observations are made; these values of z_t are then transformed for a given value of γ as follows:

$$y_1 = z_1 \qquad (5)$$

$$y_t = z_t - \gamma \sum_{i=1}^{t-1} (1 - \gamma)^{i-1} z_{t-i} \text{ for } t = 2, \ldots, n_1 + n_2.$$

by expanding this equation in terms of L, δ, μ, and Δ it can be seen that the structure of a typical y_t is

$$y_t = \mu + \Delta + (1 - \gamma)^{t-1} L + (1 - \gamma)^{t-n_1-1} \delta + a_t$$

The model, now in the form y_t, may be written as $Y = X\theta + a$ where X is defined as a N x 4 matrix of weights, θ is a 4 x 1 vector containing elements μ, Δ, L, and δ, and a is a N x 1 vector of random deviates. The equation in vector notation is as follows.

$$Y = X\theta + \alpha$$

$$
\begin{bmatrix}
y_1 \\ y_2 \\ \cdot \\ \cdot \\ \cdot \\ y_{n_1-1} \\ y_{n_1} \\ \hline y_{n_1+1} \\ \cdot \\ \cdot \\ \cdot \\ y_{n_1+n_2-1} \\ y_{n_1+n_2}
\end{bmatrix}
=
\begin{bmatrix}
1 & 0 & 1 & 0 \\
1 & 0 & (1-\gamma) & 0 \\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
1 & 0 & (1-\gamma)^{n_1-2} & 0 \\
1 & 0 & (1-\gamma)^{n_1-1} & 0 \\
\hline
1 & 1 & (1-\gamma)^{n_1} & 1 \\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot \\
1 & 1 & (1-\gamma)^{n_1+n_2-2} & (1-\gamma)^{n_2-2} \\
1 & 1 & (1-\gamma)^{n_1+n_2-1} & (1-\gamma)^{n_2-1}
\end{bmatrix}
\begin{bmatrix}
\mu \\ \Delta \\ L \\ \delta
\end{bmatrix}
+
\begin{bmatrix}
\alpha_1 \\ \alpha_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \alpha_{n_1+n_2}
\end{bmatrix}
$$

With the model now in this form, when γ is known, simple least-squares estimates of μ, Δ, L, and δ, can be determined from the familiar solution to the least-squares normal equations:

$$\hat{\theta} = \begin{bmatrix} \hat{\mu} \\ \hat{\Delta} \\ \hat{L} \\ \hat{\delta} \end{bmatrix} = (X^T X)^{-1} X^T y \tag{6}$$

The "residual variance" in fitting the model in (3) and (4) to the observations z_t is given by

$$s^2 = \left[(y - X\hat{\theta})^T (y - X\hat{\theta})\right] / (n_1 + n_2 - 4). \tag{7}$$

The following distributional statements about the estimates of the parameters follow from the assumption of normality of a_t and traditional sampling theory:

$$\frac{\hat{\mu} - \mu}{s\sqrt{c^{11}}} \sim t_{n_1+n_2-4},$$

$$\frac{\hat{\Delta} - \Delta}{s\sqrt{c^{22}}} \sim t_{n_1+n_2-4}$$

$$\frac{\hat{L} - L}{s\sqrt{c^{33}}} \sim t_{n_1+n_2-4}$$

$$\frac{\hat{\delta} - \delta}{s\sqrt{c^{44}}} \sim t_{n_1+n_2-4}, \text{ where}$$

c^{jj} is the jth diagonal element of $(X^TX)^{-1}$.

The above results follow from the linear model $Y = X\theta + \epsilon$ in which the errors, ϵ, are assumed to be normal, homoscedastic, and independent.

All of the above operations on the linear model are made for a given value of γ. When γ is unknown (as will generally be true) a Bayesian analysis using sample information about γ is used in making inferences about δ and Δ. The posterior distribution, $h(\gamma|z)$, of γ given a set of N observations and assuming a uniform prior distribution is known to within a constant of proportionality. The posterior distribution of γ assuming a uniform prior (in which case the posterior distribution is equivalent to the likelihood distribution of γ) is given to within a constant of proportionality by the following formula:

$$h(\gamma|z) \propto |X^T X|^{-\frac{1}{2}} s^{-(N-4)} . \qquad (9)$$

Illustrations of how the posterior distribution of γ in (9) is considered jointly with $\hat{\delta}$ and $\hat{\Delta}$ in making inferences about δ and Δ for the simple integrating moving average model with deterministic drift in (4) appear in Box and Tiao (1965) and Maguire and Glass (1967).

The Problem

Utilizing the model $Y = X\theta + a$, when the value of γ is known, least squares estimates of μ, L, Δ, and δ, may be determined. These estimates depend on the correctness of the assumptions of normality, homoscedasticity, and independence of the random normal variables a. The robustness (ability to stand under violations of these assumptions) of the model has been extensively studied by persons interested in the analysis of variance model. However, the IMA model's use of γ and its method of obtaining observations across equally spaced time intervals necessitates study of robustness considerations not touched upon in those studies.

The independence assumption for the Box-Tiao method may be checked through the use of autocorrelations, and in at least some cases steps may be taken to overcome violations of it (Box and Jenkins 1970, pp 176-177). Study of violations of the normality assumption, while not previously studied for this specific method was passed over in favor of variance violations, which at this time appear to have a greater probability of revealing non-robustness. In the context of the general linear model, the homogeniety of variance assumption has been studied with regard to violations across treatment levels, it has not been studied for violations of homogeniety of variance within each treatment level. Since in time-series quasi-experiments that type of violation can occur and may be a cause for concern, it is being investigated here.

Figure 1 is a graph of observations z_t versus time of observation t. The population variance of the pretreatment a_t values has been increased in equal increments from σ_0^2 at t=1 to $10\sigma_0^2$ at t=25. The population variance of all the posttreatment a_t values was held constant at σ_0^2.

The γ value used in obtaining the data for figure 1 was γ=.5. Effectively this means that half of the magnitude of each observation was stored in the system and affected the magnitude of following observations. In general this means that a_t values coming from a population with a larger variance will have a greater chance of having both a larger initial impact and consequently a larger carryover effect on following observations than would a_t values coming from a population with small variance. Consequently one would expect the slope and level of a series of observations to be more readily affected by random deviations which come from a population having a large variance. This would be particularly true if the number of observations taken was small and the value of γ large. In Figure 1 the treatment effect δ and Δ were zero, yet to the eye it appears that there may be both a change in level δ and a change in slope Δ of the graph. This study investigated the effect of several situations similar to the one noted in Figure 1.

Procedure

The four parameters, baseline L, the slope μ, and the treatment effects, change in level δ and change in slope Δ, were set at $L = 0$, $\mu = 2$, $\delta = 0$, and $\Delta = 0$. The a_t values were drawn from a pool of random normal numbers, with mean zero and variance σ_o^2, then multiplied by a value $\sqrt{c_i}$ in order to obtain a_t values with heterogeneous variances. The method then used was as follows:

1. Given the true null hypotheses $\delta = 0$, $\Delta = 0$, and $\mu = 2$, from the t-tables the $1 - \alpha$ percentile point in the t-distribution with N-4 df was determined.

2. By empirical means the actual percent of t-ratios exceeding $_{1-\alpha}t_{N-4}$ was found for each when the null hypotheses were true and the variances heterogeneous and the population normal and observations independent.

3. For each null hypothesis the nominal significance level, α, and the actual significance level were then compared.

A pseudo-random number generator FNRN (Browning 1967) was used to generate a normally distributed population pool of 3000 numbers with mean 0.00000 and variance $\sigma_o^2 = 25.108$. The normality assumption of this distribution was tested by the Kolmogorov-Smirnoff test and could not be rejected at the .20 level of significance. Since $\delta = 0$ and $\Delta = 0$, the 25 pretreatment observations $(z_1, z_2, \cdots, z_{25})$ and the

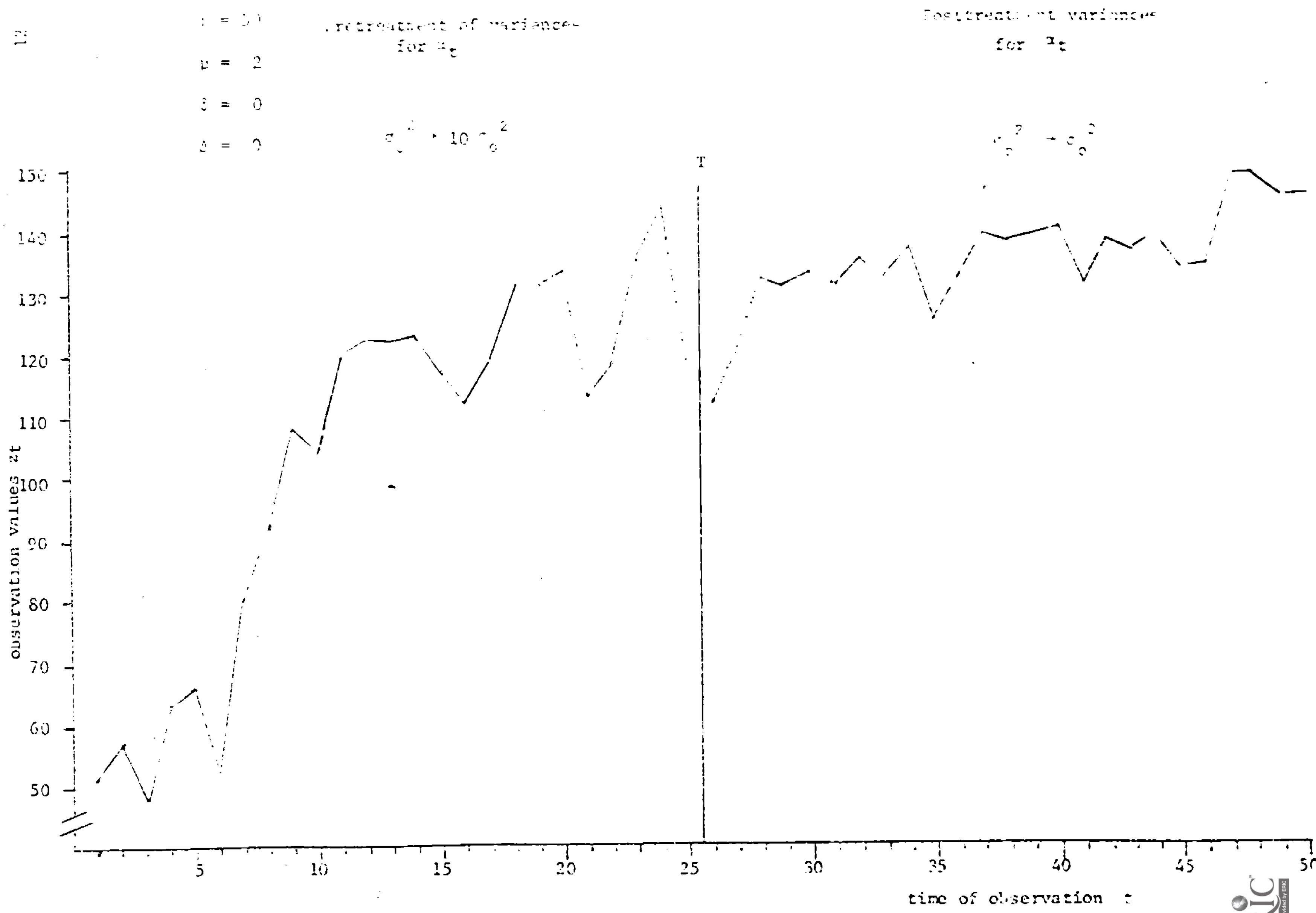

Fig. 1 Observation values z_t versus time of observation

twenty-five posttreatment observations ($z_{26}, z_{27}, \cdots, z_{50}$) necessary for each t-ratio were determined from the same formula

$$z_t = L + \gamma \sum_{i=1}^{t-1} (\mu + a_i) + \mu + a_i \text{ . Where}$$

z_t = the observed value of the process at time t.
L = 0
μ = 2
γ = .01, .50, 1.0, 1.5.

To obtain each a_i a number was drawn randomly with replacement from the pool and then multiplied by a value $\sqrt{c_i}$, to obtain an a_i value which was a random normal deviate from a population with a mean zero and variance $c_i \sigma_0^2$. t-ratios were determined for γ = .01, and this process was repeated until 1000 sets of t-ratios were formed for γ = .01. (In order to compute the t-ratios, parts of the "Computer Program for Analysis of time Series Experiment with Possible Change in Drift" by G. V Glass and T. O. Maguire was used.) This entire process was repeated for γ's of .50, 1.0, and 1.5. As was pointed out earlier in the paper, γ is not normally known, but its value is estimated from the posterior probability distribution $h(\gamma|z)$, where $0 < \gamma < 2$. The γ values .01, .05, 1.0, 1.5, used here are distributed over the range generally covered in practice by this posterior distribution.

As can be seen from Table I, nine types of variance violation and one situation in which the homogeneous variances assumption was not violated were studied. When the variance level changed it increased or decreased gradually over the length of the pretreatment or posttreatment observations. It is not expected that variance changes would necessarily occur in this smooth manner, but the situation approximates real situations closely enough to make its use feasible in this study.

TABLE I

POPULATION VARIANCE OF EACH α_t VALUE

	Pretreatment values					Posttreatment values				
RUN #	1	2	3	. . . .	25	26	27	28	. . .	50
0	σ_o^2	σ_o^2	σ_o^2	. . . .	σ_o^2	σ_o^2	σ_o^2	σ_o^2	. . .	σ_o^2
1	σ_o^2	$\frac{25}{24}\sigma_o^2$	$\frac{26}{24}\sigma_o^2$	. . .	$2\sigma_o^2$	σ_o^2	σ_o^2	σ_o^2	. . .	σ_o^2
2	σ_o^2	$\frac{28}{24}\sigma_o^2$	$\frac{32}{24}\sigma_o^2$	. . .	$5\sigma_o^2$	σ_o^2	σ_o^2	σ_o^2	. . .	σ_o^2
3	σ_o^2	$\frac{33}{24}\sigma_o^2$	$\frac{42}{24}\sigma_o^2$	. . .	$10\sigma_o^2$	σ_o^2	σ_o^2	σ_o^2	. . .	σ_o^2
4	σ_o^2	$\frac{25}{24}\sigma_o^2$	$\frac{26}{24}\sigma_o^2$	. . .	$2\sigma_o^2$	$2\sigma_o^2$	$2\sigma_o^2$	$2\sigma_o^2$	. . .	$2\sigma_o^2$
5	σ_o^2	$\frac{28}{24}\sigma_o^2$	$\frac{32}{24}\sigma_o^2$	. . .	$5\sigma_o^2$	$5\sigma_o^2$	$5\sigma_o^2$	$5\sigma_o^2$	. . .	$5\sigma_o^2$
6	σ_o^2	$\frac{33}{24}\sigma_o^2$	$\frac{42}{24}\sigma_o^2$	. . .	$10\sigma_o^2$	$10\sigma_o^2$	$10\sigma_o^2$	$10\sigma_o^2$	. . .	$10\sigma_o^2$
7	σ_o^2	$\frac{25}{24}\sigma_o^2$	$\frac{26}{24}\sigma_o^2$	. . .	$2\sigma_o^2$	$2\sigma_o^2$	$\frac{47}{24}\sigma_o^2$	$\frac{46}{24}\sigma_o^2$	. . .	σ_o^2
8	σ_o^2	$\frac{28}{24}\sigma_o^2$	$\frac{32}{24}\sigma_o^2$	. . .	$5\sigma_o^2$	$5\sigma_o^2$	$\frac{116}{24}\sigma_o^2$	$\frac{112}{24}\sigma_o^2$	. . .	σ_o^2
9	σ_o^2	$\frac{33}{24}\sigma_o^2$	$\frac{42}{24}\sigma_o^2$	. . .	$10\sigma_o^2$	$10\sigma_o^2$	$\frac{231}{24}\sigma_o^2$	$\frac{222}{24}\sigma_o^2$	. . .	σ_o^2

Results

The results for δ, change in level, are recorded in Table II, and the results for Δ, change in slope, are recorded in Table III. The γ level, and the nominal significance level, α, are at the head of each column and the actual significance levels are recorded within these columns for each of the 10 separate runs. Figure 2 is a set of 3 graphs made from the data in Table II. Each graph is for a set nominal significance level α, and shows the actual significance level of each run versus the γ values.

Interpretation of Results

Run 0 in which the assumption of homogeneity of variance was met was done as a check to insure that the computing system was functioning properly. As can be seen from Tables II and III, the differences between the nominal levels of significance and the actual levels of significance differ no more than what would be expected for a sample of size 1000.

Change in level δ: To facilitate interpretation of the data recorded in Table II, the data have been graphed in Figure 2, and the data are discussed as three separate groups A, B, and C. Each group has a common variance trend for α_i and the actual significance levels within each group maintain the same general trend. Group A consists of runs numbered 1,2, and 3, Group B consists of runs numbered 4, 5, and 6, and Group C consists of runs numbered 7, 8, and 9. General trends for each of the groups A, B, and C, are included below.

Group A had the variance trend for α_i values

$$\sigma_o^2 \rightarrow c_j\sigma_o^2,\ T,\ \sigma_o^2 \cdot c_o^2 \qquad (j = \text{the run number})$$

where c_j differs for each run j.

TABLE II

A COMPARISON OF THE NOMINAL SIGNIFICANCE LEVELS WITH THE ACTUAL SIGNIFICANCE LEVELS FOR TREATMENT EFFECT IN THE IMA MODEL, WHEN $\delta = 0$ AND THE HOMOGENEITY OF VARIANCE ASSUMPTION HAS BEEN VIOLATED.

GROUP	RUN #	PREVARIANCES Begin	PREVARIANCES End	POSTVARIANCES Begin	POSTVARIANCES End	$\gamma = .01$, $\alpha =$ 10%	5%	1%	$\gamma = .50$, $\alpha =$ 10%	5%	1%	$\gamma = 1.00$, $\alpha =$ 10%	5%	1%	$\gamma = 1.5$, $\alpha =$ 10%	5%	1%
	0	σ_o^2	σ_o^2	σ_o^2	σ_o^2	10.6	5.8	2.1	11.8	6.6	1.8	10.2	5.8	1.3	10.1	5.3	1.1
A	1	σ_o^2	$2\sigma_o^2$	σ_o^2	σ_o^2	9.3	5.5	1.8	7.9	4.0	0.9	7.5	4.0	0.4	5.5	2.8	0.4
	2	σ_o^2	$5\sigma_o^2$	σ_o^2	σ_o^2	8.2	4.7	1.1	3.0	1.2	0.1	2.7	0.4	0.0	2.8	1.0	0.0
	3	σ_o^2	$10\sigma_o^2$	σ_o^2	σ_o^2	8.0	4.4	0.8	0.9	0.2	0.0	0.4	0.0	0.0	0.4	0.0	0.0
	4	σ_o^2	$2\sigma_o^2$	$2\sigma_o^2$	$2\sigma_o^2$	10.3	6.4	2.3	14.3	7.9	2.4	10.4	6.0	1.7	11.4	5.8	1.2
B	5	σ_o^2	$5\sigma_o^2$	$5\sigma_o^2$	$5\sigma_o^2$	10.4	6.7	2.5	15.9	9.6	3.0	15.0	8.2	2.9	15.9	9.8	2.9
	6	σ_o^2	$10\sigma_o^2$	$10\sigma_o^2$	$10\sigma_o^2$	10.7	6.6	2.6	16.6	10.4	3.1	15.6	8.4	3.1	14.9	8.3	2.2
	7	σ_o^2	$2\sigma_o^2$	$2\sigma_o^2$	σ_o^2	9.8	6.0	2.1	17.0	10.3	3.4	16.5	9.2	3.5	14.2	8.1	2.5
C	8	σ_o^2	$5\sigma_o^2$	$5\sigma_o^2$	σ_o^2	10.9	6.0	2.1	21.0	14.3	5.2	21.1	13.6	5.2	21.6	14.4	5.6
	9	σ_o^2	$10\sigma_o^2$	$10\sigma_o^2$	σ_o^2	10.9	5.8	2.0	23.3	15.8	6.9	23.4	15.0	6.6	22.5	14.2	4.8

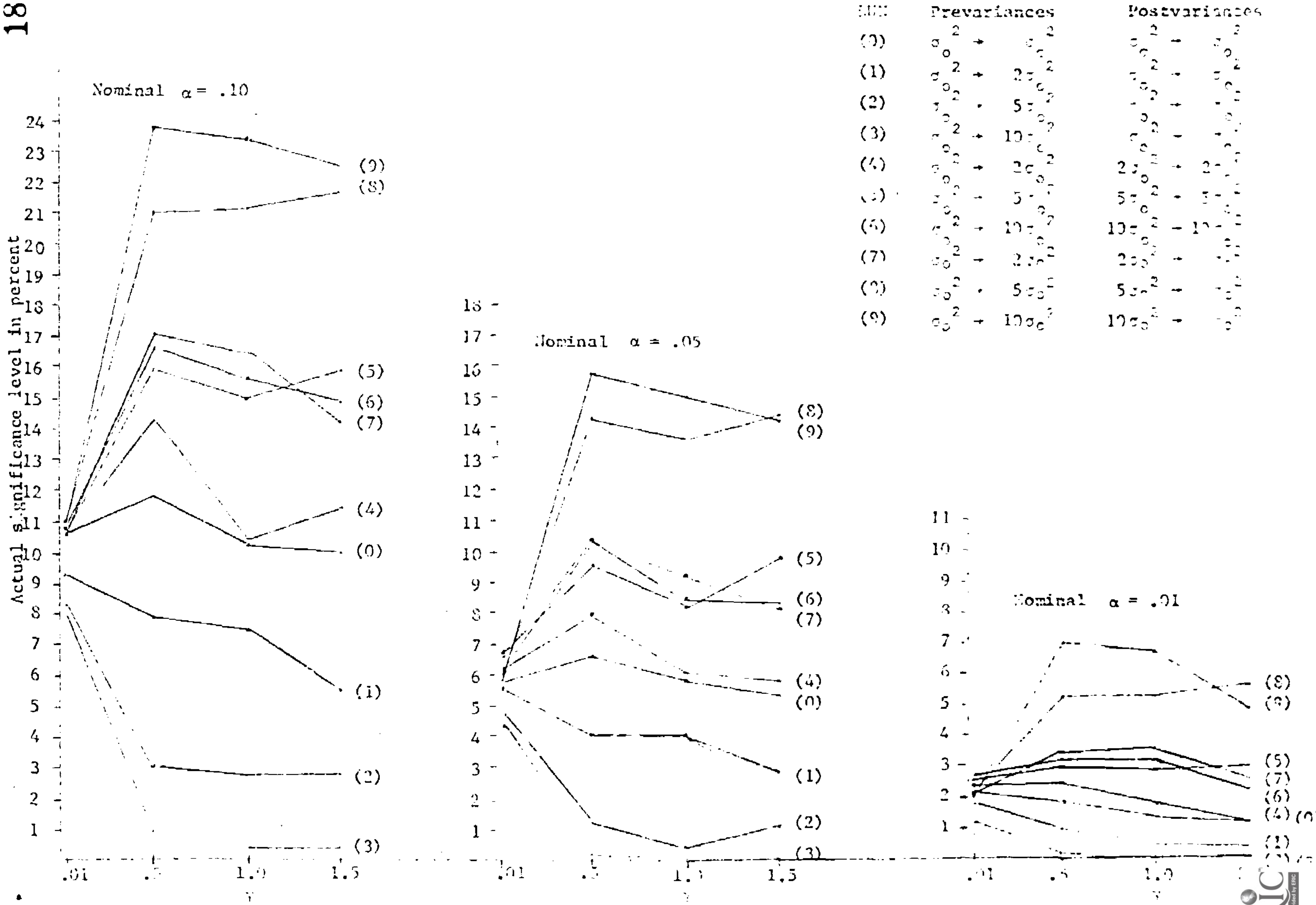

Fig. 2 The actual significance levels versus the γ values for change in level δ

TABLE III

A COMPARISON OF THE NOMINAL SIGNIFICANCE LEVELS WITH THE ACTUAL SIGNIFICANCE LEVELS FOR CHANGE IN SLOPE IN THE IMA MODEL, WHEN $\Delta = 0$ AND THE HOMOGENEITY OF VARIANCE ASSUMPTION HAS BEEN VIOLATED

GROUP	RUN #	PREVARIANCE Begin	PREVARIANCE End	POSTVARIANCE Begin	POSTVARIANCE End	$\gamma = .01$, $\alpha =$ 10%	5%	1%	$\gamma =$ [illegible], $\alpha =$ 10%	5%	1%	$\gamma = 1.00$, $\alpha =$ 10%	5%	1%	$\gamma = 1.$[illegible], $\alpha =$ 10%	5%	1%
	0	σ_0^2	σ_0^2	σ_0^2	σ_0^2	10.0	6.0	2.2	12.0	5.3	1.4	10.4	5.7	1.7	11.6	6.2	1.7
	1	σ_0^2	$2\sigma_0^2$	σ_0^2	σ_0^2	10.2	6.0	2.2	11.8	5.0	1.7	10.8	6.0	1.5	9.7	[illegible]	[illegible]
A	2	σ_0^2	$5\sigma_0^2$	σ_0^2	σ_0^2	11.3	6.2	1.9	12.5	6.6	1.5	10.9	6.2	1.9	11.3	6.2	[illegible]
	3	σ_0^2	$10\sigma_0^2$	σ_0^2	σ_0^2	12.0	6.2	1.8	12.8	7.3	1.9	10.9	6.3	1.9	11.4	6.6	[illegible]
	4	σ_0^2	$2\sigma_0^2$	$2\sigma_0^2$	$2\sigma_0^2$	9.9	5.8	2.0	12.6	5.6	1.3	12.9	7.4	2.1	9.1	4.4	1.1
B	5	σ_0^2	$5\sigma_0^2$	$5\sigma_0^2$	$5\sigma_0^2$	10.0	6.1	2.1	12.7	6.2	1.4	10.5	6.2	2.0	11.4	6.3	[illegible]
	6	σ_0^2	$10\sigma_0^2$	$10\sigma_0^2$	$10\sigma_0^2$	10.1	5.8	2.1	12.4	6.2	1.4	11.1	6.2	2.0	11.1	6.5	1.4
	7	σ_0^2	$2\sigma_0^2$	$2\sigma_0^2$	σ_0^2	10.4	5.8	2.1	11.7	5.3	1.6	10.4	6.1	1.5	9.7	4.6	[illegible]
C	8	σ_0^2	$5\sigma_0^2$	$5\sigma_0^2$	σ_0^2	10.8	5.7	2.3	11.2	5.4	1.5	11.2	6.8	1.5	11.4	6.4	[illegible]
	9	σ_0^2	$10\sigma_0^2$	$10\sigma_0^2$	σ_0^2	11.3	5.6	2.1	11.5	5.2	1.6	11.2	6.7	1.6	11.9	6.0	[illegible]

(a) As γ increased, the actual significance level decreased below α.

(b) As c_j increased in magnitude, the actual significance level decreased below α.

(c) The actual significance level was generally less than the nominal significance level.

Group B had the variance trend for a_i values

$$\sigma_o^2 \to c_j\sigma_o^2 \; , \; T \; , \; c_j\sigma_o^2 \to c_j\sigma_o^2 \qquad (j = \text{the run number})$$

where c_j differed for each run j.

(a) The actual level of significance was in all cases larger than the nominal significance level (note especially for γ = .5 and larger)

(b) The peak values of the actual significance level appear to occur for values near γ = .5.

(c) The actual level of significance increased as the magnitude of c increased.

Group C had the variance trend for a_i values

$$\sigma_o^2 \to c_j\sigma_o^2 \; , \; T \; , \; c_j\sigma_o^2 \to \sigma_o^2 \qquad (j = \text{the run number})$$

where c_j differed for each run j.

The general trend of the actual significance levels for group C followed the same pattern as for group B, and in general were more extreme in their deviations from the nominal α.

One aspect common to all runs was the robustness of the model when γ = .01. That this was to be expected can be seen from the following. When γ = 0, the IMA equation can be simplified to

$$z_t = L + \mu + \Delta + \delta + a_t$$

which is the analysis of variance model. It has previously been shown by emperical means that the analysis of variance model is robust to violations of its homogeneity of variance assumption when the treatment groups are of equal size. This study approximated the analysis of variance model for the situations in which γ = .01 (γ = .01 is a close approximation of γ = 0). Since in all cases treated here, pretreatment observations, $\underline{n}_1$ = 25, equaled $\underline{n}_2$ = 25, the number of posttreatment observations, it was expected that in the circumstances where γ = .01 the nominal and actual significance levels would closely compare.

Change in slope Δ: As can be seen from Table III the model is remarkably robust for all homogeneity of variance violations studied. None of the actual levels of significance obtained can be termed significantly different from the nominal level of significance.

Conclusions

The trends visible from the results lead to the conclusion that if possible heterogeneity of error variance is suspected then conservative nominal significance levels should be set if the IMA model is to be used in determining the effect of a treatment. This is increasingly important if the variability of the observations appears to be changing across time.

If interest centers on whether or not a treatment T has had an effect on the slope, nominal significance levels can be chosen without regard to the possible changing variability of the observations across time. As Table III shows, the model is very robust in this respect, at least with regard to all violations tested here.

FOOTNOTES

1. For a complete development of the IMA model see Box and Jenkins (1970, chapter 4).

2. The "integrated moving average model with deterministic drift" was presented by G.E.P. Box and G.M. Jenkins of pp. 33-34 of "Models for Prediction and Control, III. Linear Non-stationary Models," Technical Report No. 79. Madison: Dept. of Statistics, University of Wisconsin, July, 1966. Also see Box and Jenkins (1970).

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